Journal of Engineering Physics and Thermophysics, Vol. 82, No. 3, 2009

ANOMALIES OF THE BOUNDARY REFLECTION OF ULTRASOUND FROM A MAXWELLIAN LIQUID

D. A. Kostyuk[a] **and Yu. A. Kuzavko**[b]

UDC 621.9.08

Reflection of continuous and pulsed longitudinal and transverse acoustic waves from a dissipative medium represented by a model of Maxwellian liquid in contact with a solid halfspace is considered theoretically. The substantial dependence of the modulus and phase of the reflection coefficient on both the viscosity and time of relaxation of stresses in the Maxwellian liquid is shown. Using computer programs, the acoustic pulsed signals reflected from the interface between the media and transmitted through it have been calculated. The calculations were performed for an asymmetric shape of the signal incident on the interface; this shape corresponds to that of a real signal emitted by an ultrasonic piezoceramic transducer.

Keywords: *dissipative medium, Maxwellian liquid, longitudinal and transverse acoustic waves, spectral analysis.*

Introduction. The reflection of continuous and pulsed acoustic waves from a plane interface between media has been studied rather thoroughly [1, 2]. Earlier, we investigates, theoretically and partially experimentally, the normal (to the surface) reflection and transmission of continuous and pulsed longitudinal (LA) and transverse (TA) acoustic signals in the region of a plane interface between a solid body and a strongly dissipative medium (SDM) represented by a model of a Newtonian fluid (NF) [3–6]. Reflection and transmission coefficients differing from the classical Fresnel relations [1] and having a complex form [3] have been obtained. The parameter of dissipative losses b_2 in a Newtonian fluid determined by the viscosity and thermal conductivity of the latter in the region of frequencies $\omega \sim \omega_c$ ($\omega_c = c_2/b_2$) is responsible for the considerable dependence of R and T on frequency, whereas the spectrum of the reflected and transmitted pulsed signals undergoes noticeable changes relative to the spectrum of an incident pulsed signal [3, 4]. To describe the behavior of an extensive class of real fluids, as well as of amorphous, polymeric, and other substances there exist a rather large number of rheological models: of a Newtonian fluid, of the already mentioned Maxwellian liquid considered in the present work, Bingham and power models, a model of an elastoviscous medium, etc. [7]. Even though no universal model exists, in some cases a specific medium is quite satisfactorily described, for practical applications, by one of the above-mentioned models. Thus, the model of the Maxwellian liquid rather well characterizes soil, ground, petroleum, various oils, and polymers [7].

As a specific SDM we considered an epoxide resin (ER)–solidifier compound prepared in different weight proportions. It is one of the most accessible and efficient low-conductivity media with an appreciable viscosity varying markedly in the process of solidification. The state of the SDM qualitatively influences the amplitude and phase of both continuous and pulsed reflected and transmitted acoustic signals. Since phase-time measurements are more accurate than amplitude ones [8], they can be used to more effectively estimate sound absorption in an SDM and, by making use of the inverse problem method [9], to restore the time dynamics of the viscosity of the substance being prepared. To carry out such investigations, it is necessary to consider the characteristic features of the reflection of an acoustic signal from the interface between a solid-body acoustic line and a coat of an SDM on it (see Fig. 1). As a result of theoretical considerations, it will become possible to relate the reflection coefficient of a wave, its phase, and the spectrum of signals to such characteristics of the SDM as its viscosity and the strength of its adhesion to the acoustic line.

Longitudinal Wave. We will consider a harmonic, longitudinal acoustic wave (Fig. 1) propagating without attenuation in a solid body (medium 1) and impinging normally on a plane interface with an SDM (medium 2). The wave equation for the longitudinal acoustic wave in a Maxwellian liquid has the form [10]

---

[a]Brest State Technical University, 267 Moskovskaya Str., Brest, 224017, Belarus; [b]Poles'e Agrarian-Ecological Institute, National Academy of Sciences of Belarus, 204 Moskovskaya Str., Brest, 224000, Belarus; email: kuzavko@newmail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 82, No. 3, pp. 500–506, May–June, 2009. Original article submitted April 12, 2007; revision submitted October 29, 2007.

 1062-0125/09/8203-0496

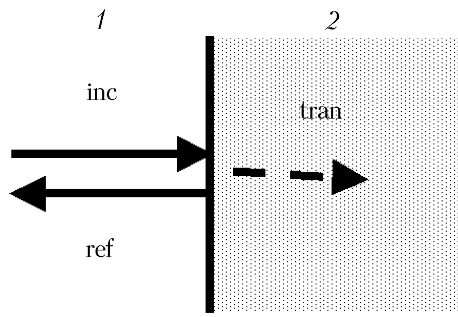

Fig. 1. Reflection and transmission of an acoustic signal.

$$\rho_2 \ddot{u}_x^{(2)} = \frac{\partial \sigma_{xx}^{(2)}}{\partial x}, \quad \sigma_{xx}^{(2)} + \tau \dot{\sigma}_{xx}^{(2)} = c_2 u_{xx}^{(2)} + b_2 u_{xx}^{(2)}, \tag{1}$$

where $\ddot{u}_x^{(2)} = \partial^2 u_x/\partial t^2$, $\dot{\sigma}_{xx}^{(2)} = \partial\sigma_{xx}^{(2)}/\partial t$, $u_{xx}^{(2)} = \partial^2 u_x/\partial x^2$, and the parameter of dissipative losses b_2 is determined by the shear and volume viscosity and by the thermal conductivity χ according to the relation [1]

$$b_2 = \frac{4}{3}\eta + \xi + \chi\left(c_v^{-1} + c_p^{-1}\right). \tag{2}$$

For a low-conductivity Maxwellian liquid to which the substances considered here relate, the third term in (2) can be neglected. Note that Eq. (1) at $b_2 = \tau = 0$ describes acoustic vibrations in a solid body.

We seek the solutions for incident, reflected, and transmitted waves in a standard form [1]:

$$u^{\text{inc}} = u_{10}^{\text{inc}} \exp\left[i\,(k_1 x - \omega t)\right], \quad u^{\text{ref}} = u_{10}^{\text{ref}} \exp\left[i\,(-k_1 x - \omega t)\right],$$

$$u^{\text{tran}} = u_{20}^{\text{tran}} \exp\left[-\alpha x + i\,(k_2 x - \omega t)\right], \tag{3}$$

where $k_1 = \omega/s_{\text{long1}}$ and $k_2 = \omega/s_{\text{long2}}$.

The boundary conditions are formulated with allowance for the continuity of displacements and stresses at the interface between the media (for a liquid medium, stress is replaced by pressure), and they have the form

$$u_x^{\text{inc}} + u_x^{\text{ref}} = u_x^{\text{tran}}, \quad c_1\left(u_{x,x}^{\text{inc}} + u_{x,x}^{\text{ref}}\right) = \sigma_{xx}^{\text{tran}}, \tag{4}$$

where $u_{x,x} = \partial u_x/\partial x$. Solutions (3) satisfy wave equations (1) and, upon substitution into Eq. (4), yield a system of linear equations that determine the amplitude coefficients of reflection $R = u_{10}^{\text{ref}}/u_{10}^{\text{inc}}$ and transmission $T = u_{20}^{\text{tran}}/u_{10}^{\text{inc}}$ (here $T = 1 + R$). We write the reflection coefficient as follows:

$$R_{\text{long}} = \frac{Z_1 f(x, \text{D}) \exp(i\psi/2) - Z_2}{Z_1 f(x, \text{D}) \exp(i\psi/2) + Z_2}, \tag{5}$$

where $f(x, \text{D}) = \left[\frac{1 + \text{D}^2}{1 + x^2}\right]^{1/4}$; $\tan\psi = \frac{x - \text{D}}{1 + \text{D}x}$; $Z_1 = \rho_1 s_{\text{long1}}$; $Z_2 = \rho_2 s_{\text{long2,0}}$; $x = \omega/\omega_c$; $\omega_c = c_2/b_2 = \rho_2 s_{\text{long2,0}}^2/b_2$ is the effective frequency that characterizes the Maxwellian liquid; $\text{D} = \omega\tau$. Note that $R_0 = (Z_1 - Z_2)/(Z_1 + Z_2)$ and $T_0 = 2Z_1/(Z_1 + Z_2)$ are the amplitude coefficients of the refection and transmission of an acoustic wave (when $\omega \to 0$) [2].

Using expression (5), we find the following relationship for the phase of a reflected signal:

$$\tan \psi_{\text{long}}^{\text{ref}} = \frac{Z_1 Z_2 (1 + \mathrm{D}^2)^{1/2} \sin (\psi/2)}{Z_1^2 (1 + \mathrm{D}^2) - Z_2^2} . \tag{6}$$

Thus, relations (5) and (6) indicate that the reflection of an acoustic wave from a Maxwellian liquid is associated with the change in its amplitude and phase relative to similar parameters of an incident wave and depends substantially on the frequency ω, the effective frequency of the Maxwellian liquid ω_c, and on the relaxation time τ. When a wave is reflected from an acoustically less dense medium ($Z_2 < Z_1$) and $\omega << \omega_c$, $\omega << \tau^{-1}$, its phase does not change at $x > \mathrm{D}$ (it is inverted at $x < \mathrm{D}$). Near $\omega \sim \omega_c$ and $\omega \sim \tau^{-1}$ the reflection coefficient has a minimum, whereas the phase shift of the reflected signal relative to the phase of the incident one undergoes a change. However, if a wave is reflected from an acoustically denser medium ($Z_2 > Z_1$) and $\omega << \omega_c$, $\omega << \tau^{-1}$, its phase is inverted at $x > \mathrm{D}$. At $\omega >> \omega_c$, $\omega >> \tau^{-1}$ we obtain $R_{\text{long}}(\omega) \to (Z_1 \mathrm{D}_c^{1/2} - Z_2)/(Z_1 \mathrm{D}_c^{1/2} + Z_2)$ and $\psi_{\text{long}}^{\text{ref}}(\omega) \to 0$ (here $\mathrm{D}_c = \omega_c \tau$).

By substituting the last of solutions (3) into Eq. (1) we find the dispersion equation of a longitudinal acoustic signal in a Maxwellian liquid:

$$k_2 = k_{2,0} \frac{1 + \mathrm{D}x}{1 + x^2} , \tag{7}$$

$$\alpha_2 = k_{2,0} \frac{|x - \mathrm{D}|}{1 + x^2} , \tag{8}$$

where $k_{2,0} = \omega/s_{\text{long}2,0}$. From relation (7) we find the phase and group velocities of a longitudinal acoustic signal:

$$v_{\text{ph}} = s_{\text{long}2,0} \frac{1 + x^2}{1 + \mathrm{D}x} , \tag{9}$$

$$v_{\text{gr}} = s_{\text{long}2,0} \frac{(1 + x^2)^2}{1 + 3x\mathrm{D} - x^2 + x^3 \mathrm{D}} . \tag{10}$$

When $x \to 0$, $v_{\text{ph}} \to s_{\text{long}2,0}$ and $v_{\text{gr}} \to s_{\text{long}2,0}$, whereas with $\omega \to \infty$ $v_{\text{ph}} \to s_{\text{long}2,0}/\mathrm{D}_c$ and $v_{\text{gr}} \to s_{\text{long}2,0}/\mathrm{D}_c$ (where $\mathrm{D}_c = \omega_c \tau$). Note the following: 1) at $\mathrm{D}_c << 1$ $v_{\text{ph}} > s_{\text{long}2,0}$, $v_{\text{gr}} > s_{\text{long}2,0}$ and $\alpha_2 \to k_{2,0}/2$ when $\omega \sim \omega_c$; 2) $|1 - \mathrm{D}_c| << 1$ $v_{\text{ph}} \to s_{\text{long}2,0}$, $v_{\text{gr}} \to s_{\text{long}2,0}$, and $\alpha_2 \to 0$; 3) $\mathrm{D}_c >> 1$, $v_{\text{ph}} < s_{\text{long}2,0}$, $v_{\text{gr}} < s_{\text{long}2,0}$, and $\alpha_2 \to k_{2,0} \mathrm{D}_c/2 >> k_{2,0}$ when $\omega \sim \omega_c$. The condition $\mathrm{D}_c = 1$ means the coincidence of the effective frequency ω_c with the relaxational frequency τ^{-1}, and it is precisely at that time that a weakly attenuating longitudinal acoustic wave exists in a Maxwellian liquid.

For completeness of consideration, we also give expressions of the transmission coefficients $T_{\text{long}}(\omega)$ and of its phase $\psi_{\text{long}}^{\text{tran}}(\omega)$ for a longitudinal acoustic wave in a Maxwellian liquid:

$$T_{\text{long}} = \frac{2 Z_1 f(x, \mathrm{D}) \exp (i\psi/2)}{Z_1 f(x, \mathrm{D}) \exp (i\psi/2) + Z_2} , \tag{11}$$

$$\psi_{\text{long}}^{\text{tran}} = \frac{\psi}{2} - \varphi , \tag{12}$$

where $\tan \varphi = \left[Z_1 f(x, \mathrm{D}) \sin \frac{\psi}{2} \right] \Big/ \left[Z_2 + Z_1 f(x, \mathrm{D}) \cos \frac{\psi}{2} \right]$.

Figure 2a characterizes the frequency dependences of modules and phases of the amplitude reflection coefficients in the cases of reflection without inversion ($R_0 > 0$, aluminum–ER) and with inversion ($R_0 < 0$, plastic–ER). The

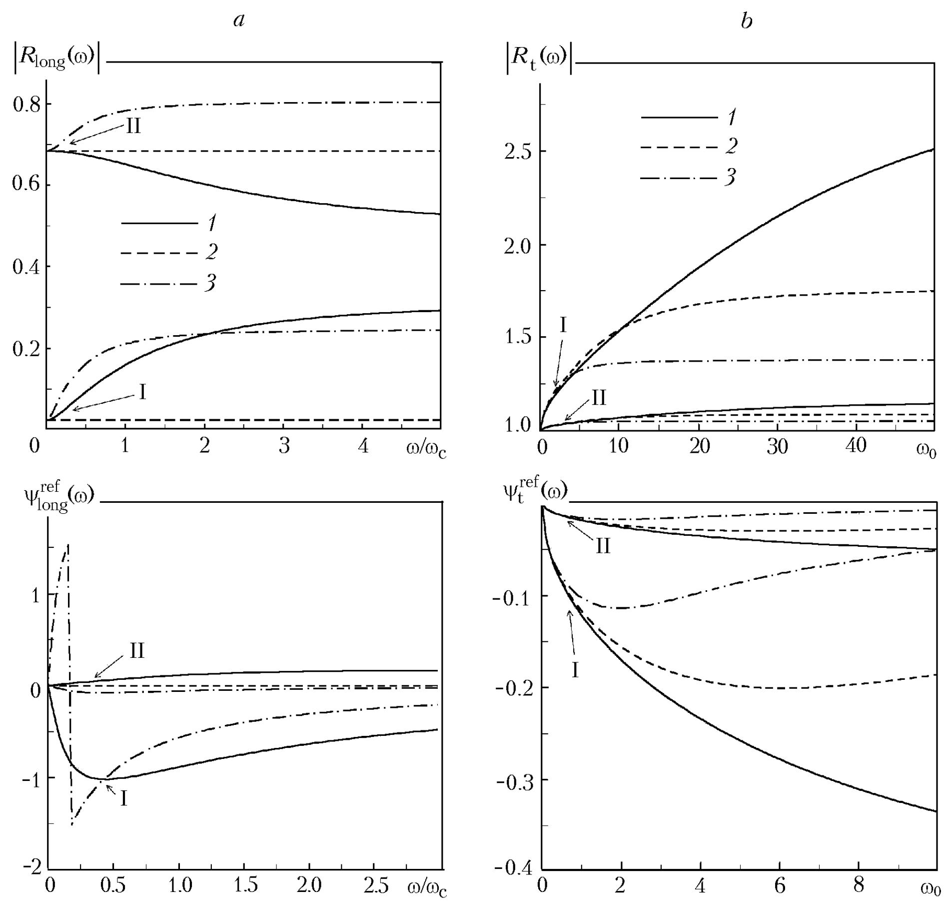

Fig. 2. Frequency dependences of the amplitude reflection coefficients of a longitudinal acoustic signal (a) and a transverse acoustic signal (b) on the interface between media (I, plastic–ER compound; II, aluminum–ER compound): a) 1, $\tau\omega_c$ = 0.3; 2, 1.0; 3, 3.0; b) 1, τ = 0.03 μsec; 2, 0.1; 3, 0.3.

dependences were constructed with the use of the following material parameters: for the plastic $Z_1 = 3.1\cdot10^6$ kg/(m^2·sec), s_{long1} = 2.7 km/sec; for aluminum $Z_1 = 17.33\cdot10^6$ kg/(m^2·sec), s_{long1} = 6.42 km/sec; for epoxide resin $Z_2 = 3.25\cdot10^6$ kg/(m^2·sec), $s_{long2,0}$ = 2.68 km/sec, $\omega_c = 2\pi\cdot10^7$ Hz (the data are given for the solid phase of ER).

From the practical point of view, reflected signals are more informative than weakened ones that passed through the SDM layer. In addition to the amplitude reflection coefficient there is no difficulty in determining the reflection coefficients from the velocity, pressure, and intensity [10]. The relationship between the reflection coefficients is of general character, and it is also valid for a transverse acoustic waves and any SDM represented by the models of a Newtonian fluid, Maxwellian liquid, etc.

Transverse Wave. The wave equation for a transverse acoustic signal in a Maxwellian liquid has the form [10]

$$\rho_2 \ddot{u}_y^{(2)} = \frac{\partial u_{yx}^{(2)}}{\partial x}, \quad \sigma_{yx}^{(2)} + \tau \dot{\sigma}_{yx}^{(2)} = \eta \dot{u}_{yx}^{(2)}, \tag{13}$$

where $\ddot{u}_y^{(2)} = \partial^2 u_y/\partial t^2$; $\dot{\sigma}_{yx}^2 = \partial\sigma_{yx}^{(2)}/\partial t$; $\dot{u}_{yx}^{(2)} = \partial^3 u_y/\partial x^2 \partial t$.

The phase velocity of a transverse acoustic signal is defined by the relation

$$v_{ph} = \left(\frac{2\eta\tau}{\rho_2}\right)^{1/2} \frac{1}{D\,[1 + (1 + D^2)^{1/2}]^{1/2}}, \tag{14}$$

whereas the group velocity $v_{gr} > v_{ph}$ is expressed as

$$v_{gr} = \left(\frac{2\eta\tau}{\rho_2}\right)^{1/2} \frac{(1+D^2)^{1/2}\,[1+(1+D^2)^{1/2}]^{1/2}}{1+\frac{3}{2}D^2+(1+D^2)^{1/2}}\,. \tag{15}$$

Expressions (14) and (15) result from the complexity of the wave number k_2, the real and imaginary parts of which are determined as follows:

$$\mathrm{Re}\,(k_2) = \left(\frac{\rho_2}{2\eta}\right)^{1/2} D^{1/2}\,[1+(1+D^2)^{1/2}]^{1/2}\,\omega^{1/2}\,, \tag{16}$$

$$\mathrm{Im}\,(k_2) = \left(\frac{\rho_2}{2\eta\tau}\right)^{1/2} [1+(1+D^2)^{1/2}]^{-1/2}\,. \tag{17}$$

Here, $\mathrm{Im}(k_2)$ represents the magnitude of absorption of a transverse acoustic signal in a Maxwellian liquid, the limiting values of which are $\mathrm{Im}\,[k_2(\omega \to 0)] \to (\rho_2/4\eta\tau)^{1/2}$ and $\mathrm{Im}\,[k_2(\omega \to \infty)] \sim (\rho_2/2\eta\tau)^{1/2}D^{-1/2} \to 0$. The limiting values of the phase and group velocities are equal between themselves: $v_{ph}(\omega \to \infty) = v_{gr}(\omega \to \infty) = 0$.

The reflection coefficient for a transverse acoustic signal has the form

$$R_t = \frac{Z_1 + a}{Z_1 - a}\,, \tag{18}$$

where $a = i\eta k_2/(1 - iD)$, $k_2 = (i\omega\rho_2(1 - iD)/\eta)^{1/2}$. At low frequencies $R_t \to 1$, $\psi_t^{ref} \to 0$ and at high frequencies $R_t \to (Z_1 - (\rho_2\eta/\tau)^{1/2})/\ (Z_1 - (\rho_2\eta/\tau)^{1/2})$, $\psi_t^{ref} \to 0(\pi)$.

The behavior of the reflection coefficient is shown in Fig. 2b. The dependences were constructed with the use of the following material parameters: for plastic $Z_1 = 1.26\cdot10^6$ kg/(m^2·sec), $s_{t1} = 1.1$ km/sec; for aluminum $Z_1 = 8.21\cdot10^6$ kg/(m^2·sec), $s_{t1} = 3.04$ km/sec; for epoxide resin $Z_2 = 1.39\cdot10^6$ kg/(m^2·sec), $s_{t2,0} = 1.15$ km/sec, $\eta = 500$ Pa·sec (just as in the case of a longitudinal acoustic signal, the data are given for the solid phase of epoxide resin).

Computer Simulation. We will consider the case of reflection of a pulsed acoustic signal that is closest to the signal emitted by an ultrasonic piesoceramic transducer. It can be presented in the form

$$u^{inc}(x=0, t) = A_1 \exp\left(\frac{\Gamma_1 t}{T}\right)\exp\left(i2\pi\frac{t}{T}\right)[\theta(t+\tau_1) - \theta(t)]$$
$$+ A_2 \exp\left(-\frac{\Gamma_2 t}{T}\right)\exp\left(-i2\pi\frac{t}{T}\right)[\theta(t) - \theta(t-\tau_2)] \tag{19}$$

with asymmetric distribution of vibrations in a pulse (see the inset in Fig. 3a). Here, the parameters Γ_1 and Γ_2 characterize the envelope of the pulse respectively before and after the maximum of its sweep; $T = 2\pi/\omega_0$; $\tau_1 = n_1T/4$, $\tau_2 = n_2T/4$; n_1 and n_2 are integers that influence the shape of the curve enveloping integers, the sum of which is equal to the number of radiated pulse periods, A_1 and A_2 are numbers that influence the shape of the enveloping curve, the sum of which is equal to u_{10}^{inc}.

Proceeding from the given function $R(\omega)$ and using expressions (19), as well as applying the direct and inverse Fourier transformation, we calculated the shape of a reflected signal on a computer. The results of calculations in the form of the sweep curves of the pulse and phase signal are presented in Fig. 3 for the longitudinal acoustic signal (a) and transverse acoustic signal (b); they indicate the substantial dependence of the amplitude and phase of the reflected signal on the frequency ω_0 of the principal harmonic of the emitted signal, as well as on the elastic and dissipative characteristics of a Maxwellian liquid. The shift of the phase of a reflected pulse is understood more generally

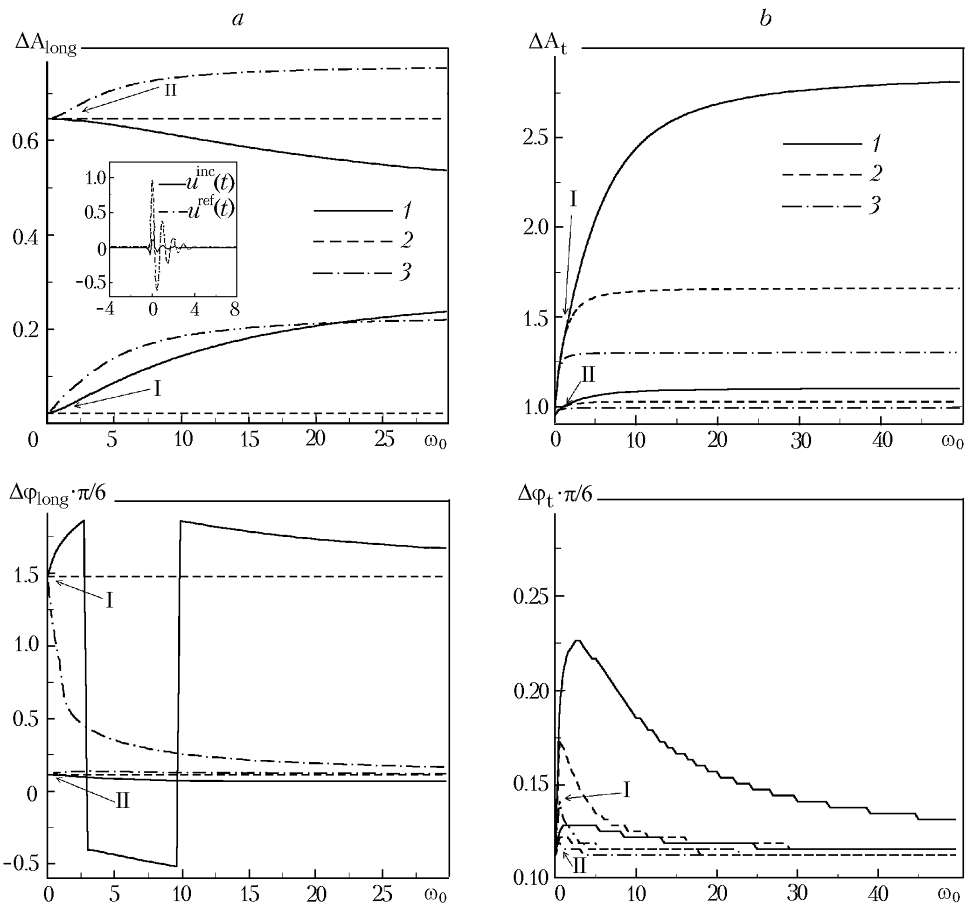

Fig. 3. Frequency dependences of the sweep of a reflected pulse and of its phase shift of a longitudinal (a) and transverse (b) acoustic signal on the interface between media (I, plastic–ER compound; II, aluminum–ER compound) (the shape of the incident (n_1 = 3 and n_2 = 17) and reflected pulses is shown on the inset): a) 1, $\tau\omega_c$ = 0.3; 2, 1.0; 3, 3.0; b) 1, τ = 0.03 μsec; 2, 0.1; 3, 0.3.

than is the case for continuous vibrations, viz., as displacement of the intersection with the time axis of the emitted and reflected pulses. The Fourier transformation applied to the emitted $u^{\text{inc}}(t)$ and reflected $u^{\text{ref}}(t)$ pulses yields their spectra that differ from one another.

For the signal (19) the spectrum has the form

$$F^{\text{inc}}(\omega) = \frac{A_1}{2\pi} \frac{1}{\frac{\Gamma_1}{T} + i(\omega + \omega_0)} \left[1 - \exp(-\Gamma_1 n_1) \exp\left(-i2\pi n_1 \frac{\omega}{\omega_0} \right) \right]$$

$$+ \frac{A_2}{2\pi} \frac{1}{\frac{\Gamma_2}{T} - i(\omega + \omega_0)} \left[1 - \exp(-\Gamma_2 n_2) \exp\left(i2\pi n_2 \frac{\omega}{\omega_0} \right) \right]. \quad (20)$$

Assuming that $A_1 = A_2 = u_{10}^{\text{inc}}/2$, $\Gamma_1 = \Gamma_2$, $n_1 = n_2$, we obtain the spectrum of a symmetric signal. However, the spectrum and shape of transformed (reflected and transmitted) pulsed signals cannot be found analytically because of the complex frequency-dependent form of R and T. Note that the software used makes it possible to reveal the characteristic features of reflection for any forms of the pulses being emitted.

Conclusions. The developed theory of normal reflection of longitudinal and transverse acoustic continuous and pulsed signals from a plane interface between a solid body and a Maxwellian liquid is applicable for the analysis of dissipative and relaxational processes proceeding in an extensive group of liquid, amorphous, and organic substances. It is evident that the use of the method of inverse problem for the restoration of the temporal dynamics of SDM viscosity allows one to determine the viscosity (internal friction) of substances for which direct measurements are difficult or impossible.

It should be expected that such means can be used for investigating a large group of substances undergoing phase, structural, aggregate, and chemical conversions, as well as being under the influence of strong temperature, electromagnetic and other factors that alter their material parameters (density, elasticity, velocity and absorption of sound, plasticity and fluidity, viscosity (internal friction), thermal conductivity). Evidently from such measurements it is possible to diagnose the qualitative and quantitative characteristics of mechanical engineering products and radioelectronic components, in the process of the fabrication of which local concentrated regions of a stressed state in a material subjected to a strong heating appear that provide the possibility of appearance of structural and other transformations. As a result, it becomes possible to judge the state of this or other technological product.

The work was carried out with financial support from the Belarusian Basic Research Foundation (grant T06M-227), State Program of Applied Investigations "Reduction of the Risks of Extreme Situations" (assignment 15).

NOTATION

b_2, parameter of dissipative losses in a SDM, Pa·sec; c_2, elasticity modules of a SDM, J/m^3; c_p, c_v, heat capacities of a medium at a constant pressure and volume, J/(kg·K); D, Deborah number; i, imaginary unit; k_1, k_2, wave numbers for media 1 and 2, m^{-1}; R, amplitude coefficient of sound reflection; R_0, amplitude frequency-independent reflection coefficient; $R_{\rm long}$, $R_{\rm t}$, amplitude coefficients of reflection of a longitudinal and transverse sound; $s_{\rm long1}$, $s_{\rm long2}$, velocity of longitudinal sound in media 1 and 2, m/sec; $s_{\rm t1}$, velocity of a transverse sound in medium 1, m/sec; $s_{\rm long2,0}$, $s_{\rm t2,0}$, velocities of a longitudinal and transverse sound in a SDM in the absence of dissipation (at $\omega = 0$), m/sec; T, amplitude coefficient of sound transmission; T_0, amplitude frequency-independent transmission coefficient; $T_{\rm long}$, amplitude coefficient of transmission of a longitudinal sound; t, time, sec; $u^{\rm inc}$, $u^{\rm ref}$, and $u^{\rm tran}$, emitted, reflected, and transmitted acoustic signals, m; $u_{10}^{\rm inc}$, $u_{10}^{\rm ref}$, and $u_{20}^{\rm tran}$, amplitudes of the emitted, reflected, and transmitted acoustic signals, m; u_x, component of longitudinal displacement in a wave, m; u_y, component of elastic displacement in a transverse wave, m; $v_{\rm gr}$, group velocity, m/sec; $v_{\rm ph}$, phase velocity, m/sec; x, coordinate over the abscissa axis, m; Z_1, Z_2, acoustic impedances of media 1 and 2 on the assumption of the absence of dissipation, kg/(m^2·sec); α, coefficient of sound absorption, 1/m; α_2, coefficient of sound absorption in a SDM, 1/m; $\Delta A_{\rm long}$, $\Delta A_{\rm t}$, sweep of longitudinal and transverse pulses, m; $\Delta\varphi_{\rm long}$, $\Delta\varphi_{\rm t}$, phase shift of a longitudinal and transverse pulses, rad; η, shear viscosity, Pa·sec; ξ, volume viscosity, Pa·sec; θ, Heavyside function; ρ_1, ρ_2, densities of media 1 and 2, kg/m^3; σ_{xx}, σ_{yx}, stress of the longitudinal and transverse sound, Pa; τ, time of relaxation, sec; χ, thermal conductivity, W·m·sec/K; $\psi_{\rm long}^{\rm ref}$, $\psi_{\rm t}^{\rm ref}$, phases of a longitudinal and transverse reflected waves, rad; $\psi_{\rm long}^{\rm tran}$, phase of a transmitted longitudinal wave, rad; ω, cyclic frequency, Hz; $\omega_{\rm c}$, cyclic frequency characterizing SDM, Hz; ω_0, frequency of the principal harmonics of pulsed signal, Hz. Indices: 0, absence of dissipation; 1, solid acoustic line (medium 1); 2, SDM (medium 2); c, characteristic; inc, incident wave; long, longitudinal sound; ref, reflected wave; t, transverse sound; tran, transmitted wave; ph, phase; gr, group.

REFERENCES

1. E. D'elesan and D. Ruaie, *Elastic Waves in Solids* [Russian translation], Nauka, Moscow (1982).
2. M. B. Vinogradova, O. V. Rudenko, and A. P. Sukhorukov, *Theory of Waves* [in Russian], Nauka, Moscow (1990).
3. D. A. Kostyuk and Yu. A. Kuzavko, Anomalies in the threshold reflection of an ultrasound from a dissipative medium, *Pis'ma Zh. Tekh. Fiz.*, **27**, Issue 23, 31–40 (2001).
4. D. A. Kostyuk and Yu. A. Kuzavko, Characteristic features of threshold reflection of transverse waves from a dissipative medium, *Vestnik BGTU*, *Mashinostroenie*, *Avtomatizatsiya*, *ÉVM*, No. 4, 48–51 (2000).

5. D. A. Kostyuk and Yu. A. Kuzavko, Anomalous reflection of a longitudinal ultrasonic wave from a strong dissipative medium, *Inzh.-Fiz. Zh.*, **77**, No. 5, 161–169 (2004).
6. M. Zh. Burlibaev, A. A. Volchek, D. A. Kostyuk, and Yu. A. Kuzavko, Application of the acoustics of strongly dissipative media to the solution of industrial, agricultural, and ecological problems: state-of-the-art, perspectives, and forecasting, *Gidrometeorol. Ékol.*, Nos. 1–2, 141–154 (2001).
7. Z. P. Shul'man and B. M. Berkovskii, *Boundary Layer of Non-Newtonian Fluids* [in Russian], Nauka i Tekhnika, Minsk (1966).
8. P. V. Novitskii, *Principles of the Information Theory of Measuring Devices* [in Russian], Énergiya, Leningrad (1968).
9. V. Yu. Zavadskii, *Finite-Differences Method in the Wave Problems of Acoustics* [in Russian], Nauka, Moscow (1982).
10. A. F. Kozak, D. A. Kostyuk, Yu. A. Kuzavko, and L. N. Nikolayuk, Reflection of acoustic waves from a boundary with a Maxwellian liquid, in: *Abstracts of papers submitted to the 44th Int. Conf. "Urgent Problems of Strength,"* Vologda (2005), p. 147.